QN	T.Y.B.Sc.(Mathematics) Subject: MTH-502: Real Analysis Question Bank	Ans
1)	If the function $f(x) = x^2$ is integrable on $[0,a]$ then $\int_0^a f(x)\mathrm{d}x = \ldots$ (A) 0 (B) a (C) $\frac{a^3}{3}$ (D) a^2	**(C)**
2)	If $f(x) = \begin{cases} 1, & if\ x\ is\ rational \\ -1, & if\ x\ is\ irrational \end{cases}$ then (A) $f \in R[0,1]$ (B) $f \notin R[0,1]$ (C) $f(x)$ is unbounded (D) None of these	**(B)**
3)	If f_1 and f_2 are bounded and integrable functions on $[a, b]$ then $(f_1 + f_2)$ is …. (A) bounded and integrable functions on $[a, b]$ (B) Not bounded and integrable functions on $[a, b]$ (C) bounded but not integrable functions on $[a, b]$ (D) Neither bounded nor integrable functions on $[a, b]$	**(A)**
4)	If f_1 and f_2 are bounded and integrable functions on $[a, b]$ then $(f_1 - f_2)$ is …. (A) bounded and integrable functions on $[a, b]$ (B) Not bounded and integrable functions on $[a, b]$ (C) bounded but not integrable functions on $[a, b]$ (D) Neither bounded nor integrable functions on $[a, b]$	**(A)**
5)	If f_1 and f_2 are bounded and integrable functions on $[a, b]$ then $(f_1 * f_2)$ is …. (A) bounded and integrable functions on $[a, b]$ (B) Not bounded and integrable functions on $[a, b]$ (C) bounded but not integrable functions on $[a, b]$	**(A)**

	(D) Neither bounded nor integrable functions on $[a, b]$	
6)	If f is continuous and integrable on $[a, b]$, then there exists a number c between a and b such that…. (A) $\int_a^b f\ \mathrm{d}x < (b-c)f(c)$ (B) $\int_a^b f\ \mathrm{d}x = (b-c)f(c)$ (C) $\int_a^b f\ \mathrm{d}x > (b-c)f(c)$ (D) None of these	**(B)**
7)	If f is bounded and integrable on $[a, b]$ and k is a number such that $\lvert f\ (x)\rvert \leq k$ for all $x \in [a, b]$, then…. (A) $\left\lvert\int_a^b f\ \mathrm{d}x\right\rvert < k\lvert b-a\rvert$ (B) $\left\lvert\int_a^b f\ \mathrm{d}x\right\rvert > k\lvert b-a\rvert$ (C) $\left\lvert\int_a^b f\ \mathrm{d}x\right\rvert \leq k\lvert b-a\rvert$ (D) None of these	**(C)**
8)	If P_1 and P_2 are partitions of $[a, b]$ then a partition P is a common refinement of P_1 and P_2 if…. (A) $P \subseteq P_1 \cup P_2$ (B) $P \supseteq P_1 \cup P_2$ (C) $P = P_1 \cup P_2$ (D) None of these	**(C)**
9)	If p^* is a refinement of a partition P then for a bounded function f … (A) $L(p^*, f) > L(P, f)$ (B) $L(p^*, f) < L(P, f)$ (C) $L(p^*, f) \geq L(P, f)$ (D) None of these	**(C)**
10)	If p^* is a refinement of a partition P then for a bounded function f … (A) $U(p^*, f) \leq U(P, f)$ (B) $U(p^*, f) > U(P, f)$	**(A)**

	(C) $U(p^*, f) < U(P, f)$ (D) None of these	
11)	The oscillation of a bounded function f in $[a, b]$ is… (A) $Sup\ \{\lvert f(\alpha) - f(\beta)\rvert : \alpha, \beta \epsilon [a, b]\}$ (B) $Inf\ \{\lvert f(\alpha) - f(\beta)\rvert : \alpha, \beta \epsilon [a, b]\}$ (C) $Sup\ \{\lvert f(x)\rvert : x \epsilon [a, b]\}$ (D) $Inf\ \{\lvert f(x)\rvert : x \epsilon [a, b]\}$	**(A)**
12)	If P is a any partition of $[a, b]$, then for a bounded function f …. (A) $U(P, f) \geq L(P, f)$ (B) $U(P, f) \leq L(P, f)$ (C) $U(P, f) \neq L(P, f)$ (D) None of these	**(A)**
13)	If $f(x) = k, \forall x \in [a, b]$ where k is constant then $U(P, f)$..... (A) $= k(b - a)$ (B) $> k(b - a)$ (C) $< k(b - a)$ (D) None of these	**(A)**
14)	If $f(x) = k, \forall x \in [a, b]$ where k is constant then $L(P, f)$..... (A) $> k(b - a)$ (B) $= k(b - a)$ (C) $< k(b - a)$ (D) None of these	**(B)**
15)	If f is bounded and integrable on $[a, b]$, then f is… (A) not integrable on $[a, b]$ (B) integrable on $[a, b]$ (C) constant on $[a, b]$ (D) None of these	**(B)**

16)	If f is bounded and integrable on $[a, b]$, then f^2 is… (A) not integrable on $[a, b]$ (B) integrable on $[a, b]$ (C) unbounded on $[a, b]$ (D) None of these	**(B)**
17)	$\int_0^b f\ (x)\mathrm{d}x = ...$ (A) $\lim\limits_{n\to\infty} \sum_{r=1}^{n} hf(a + rh)$ (B) $\lim\limits_{n\to 0} \sum_{r=1}^{n} hf(a + rh)$ (C) $\lim\limits_{n\to\infty} \sum_{r=1}^{\infty} hf(a + rh)$ (D) $\lim\limits_{n\to 0} \sum_{r=1}^{\infty} hf(a + rh)$	**(A)**
18)	If $f(x) = 3x + 2$, then $\int_1^2 f\ (x)\mathrm{d}x = ...$ (A) $\frac{11}{2}$ (B) $\frac{10}{3}$ (C) $\frac{3}{2}$ (D) $\frac{1}{2}$	**(A)**
19)	$$\lim_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\cdots+\frac{n}{2n}\right]=\cdots$$ (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$ (C) π (D) $\frac{\pi}{4}$	**(D)**
20)	$\lim\limits_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{n+n}\right]=\cdots$ (A) $\log n$ (B) $\log 6$ (C) $\log 2$ (D) $\log 4$	**(C)**
21)	$\lim\limits_{n\to\infty}\left[e^{\frac{1}{n}}+e^{\frac{2}{n}}+e^{\frac{3}{n}}+\cdots+e^{\frac{n}{n}}\right]=\cdots$ (A) e (B) e + 1	**(C)**

	(C) e -1 (D) 0	
22)	$\lim_{n\to\infty}\left(\frac{n^n}{n!}\right)^{\frac{1}{n}} = \cdots$ (A) $\frac{1}{e}$ (B) 0 (C) 1 (D) e	**(D)**
23)	The function $(x) = \frac{1}{2^n}$; $\frac{1}{2^{n+1}} < x \le \frac{1}{2^n}$; (n = 0,1,2,…) and $f(0) = 0$ has ….points of discontinuity. (A) finite (B) one (C) infinite (D) zero	**(C)**
24)	If f and g are integrable on $[a, b]$ and g keeps the same sign over $[a, b]$, then there exists a number μ lying between the bounds of f such that $\int_a^b f\,(x)\mathrm{d}x =$ ... (A) $> \mu \int_a^b g\mathrm{d}x$ (B) $< \mu \int_a^b g\mathrm{d}x$ (C) $\mu + \int_a^b g\mathrm{d}x$ (D) $= \mu \int_a^b g\mathrm{d}x$	**(D)**
25)	If $\int_a^b f\mathrm{d}x$ and $\int_a^b g\mathrm{d}x$ both exists and f is monotone on $[a, b]$, then there exists $\xi \in [a, b]$ such that (A) $\int_a^b fg\mathrm{d}x = f(a)\int_a^{\xi} g\mathrm{d}x + f(b)\int_{\xi}^b g\mathrm{d}x$ (B) $\int_a^b fg\mathrm{d}x = g(a)\int_a^{\xi} f\mathrm{d}x + f(b)\int_{\xi}^b g\mathrm{d}x$ (C) $\int_a^b fg\mathrm{d}x = g(a)\int_a^{\xi} f\mathrm{d}x + g(b)\int_{\xi}^b f\mathrm{d}x$ (D) $\int_a^b fg\mathrm{d}x = f(a)\int_a^{\mathrm{b}} g\mathrm{d}x + f(b)\int_{\mathrm{a}}^b g\mathrm{d}x$	**(A)**
26)	If $f(x) = x$ and $g(x) = e^x$ in $[-1,1]$ then $\int_{-1}^{1} f(x)\mathrm{g(x)}\mathrm{d}x =$ ... (A) $\frac{e}{2}$ (B) $\frac{2}{e}$ (C) $2e$ (D) $\frac{4}{e}$	**(B)**
27)	If $0 < a < b$, then $\left\|\int_a^b \sin x^2\,\mathrm{d}x\right\|$ is …	**(C)**

	(A) $\geq \frac{1}{a}$ (B) $\leq \frac{2}{a}$ (C) $\leq \frac{1}{a}$ (D) $\geq \frac{2}{a}$	
28)	For $b \in [0, 1]$, $\int_0^1 \frac{\sin \pi x}{1+x^2} \mathrm{d}x = ...$ (A) $\frac{\pi}{2} \sin \pi b$ (B) $\frac{2}{\pi} \sin \pi b$ (C) $\frac{2}{b^2+1}$ (D) $\frac{\pi}{4} \sin \pi b$	**(D)**
29)	For $\xi \in [a, b]$; $\int_0^1 \frac{\sin \pi x}{1+x^4} \mathrm{d}x = ...$ (A) $\frac{2}{\pi(\xi^2+1)}$ (B) $\frac{2}{\pi(\xi^4+1)}$ (C) $\frac{4}{\pi(\xi^2+1)}$ (D) None of these	**(B)**
30)	If $n > 0$, then $\int_0^{\frac{x}{2}} \cos^n x \, \mathrm{d}x = \cdots$ (A) proper (B) improper (C) divergent (D) none of these	**(B)**
31)	For the integral $\int_0^{\frac{x}{2}} \frac{1}{\sqrt{\tan x}} \mathrm{d}x = \cdots$ the points of infinite discontinuity is…. (A) $\frac{\pi}{4}$ (B) 0 (C) Both (A) and (B) (D) None of these	**(B)**
32)	The integral $\int_0^{\infty} \frac{x^2}{\sqrt{x^5+1}} \mathrm{d}x = \cdots$ (A) convergent (B) divergent (C) exists (D) None of these	**(B)**
33)	The integral $\int_0^{\infty} \frac{\sin x}{x^p} \mathrm{d}x$ converges absolutely if… (A) $p < 1$ (B) $p = 0$ (C) $p > 1$ (D) None of these	**(C)**
34)	The integral $\int_a^{x} \sin x \, \mathrm{d}x$ is …………for $X \geq a$.	**(A)**

	(A) bounded (B) not bounded (C) unbounded (D) None of these	
35)	The differential equation of the form $(1-x^2)y''-2xy'+n(n+1)y=0$; where n a positive integer is called…. (A) Cauchy-Riemann Equation (B) Bernoulli's equation (C) Clairaut's Equation (D) Legendre's equation	**(D)**
36)	If A and B are arbitrary constants, then the general solution of the equation $(1-x^2)y''-2xy'+n(n+1)y=0$ is given by…. (A) $y=AP_n(x)+BQ_n(x)$ (B) $y=A+B$ (C) $y=0$ (D) $y=1$	**(A)**
37)	The expansion of x^3 in terms of Legendre's polynomial is… (A) $2P_3(x)+3P_1(x)$ (B) $\frac{2}{5}P_3(x)+\frac{3}{5}P_1(x)$ (C) $\frac{2}{5}P_3(x)-\frac{3}{5}P_1(x)$ (D) None of these	**(B)**
38)	The integral $\int_a^\infty f\mathrm{d}x$ converges at ∞ if and only if for every $\varepsilon>0$ there corresponds a positive number x_0 such that for all $x_1,x_2\geq x_0$, (A) $\left\|\int_{x_1}^{x_2} f(x)\mathrm{d}x\right\|<\varepsilon$ (B) $\left\|\int_a^\infty f(x)\mathrm{d}x\right\|<\varepsilon$ (C) $\left\|\int_{a+\lambda_1}^{a+\lambda_2} f(x)\mathrm{d}x\right\|<\varepsilon$ (D) None of these	**(A)**
39)	$P_{2m}(0)=\cdots$ (A) 0 (B) 1 (C) $\frac{(-1)^m(2m)!}{2^{2m}(m!)^2}$ (D) 2m	**(C)**

40)	$\left\lceil\frac{1}{2}\right.$ (A) π (B) $\sqrt{\pi}$ (C) 1 (D) $\sqrt{\frac{\pi}{2}}$	(B)
41)	For the partition $P=\{1, 3, 5, 8\}$ of $[1, 8]$ the length $\Delta x_1 = ...$ (A) 2 (B) 1 (C) 3 (D) none of these	(A)
42)	For any partition P the norm $\mu(P)\rightarrow$.... as $n\rightarrow\infty$ (A) 1 (B) 0 (C) ∞ (D) $-\infty$	(B)
43)	The partition P of $[a, b]$ is…. (A) Finite subset of $[a, b]$ (B) Infinite subset of $[a, b]$ (C) Both (A) and (B) (D) None of these	(A)
44)	The formula for $L\,(P, f)$ is…. (A) $\sum_{i=1}^{\infty} m_i\,\Delta x_i$ (B) $\sum_{i=1}^{n} M_i\,\Delta x_i$ (C) $\sum_{i=1}^{n} m_i\,\Delta x_i$ (D) $\sum_{i=1}^{\infty} M_i\,\Delta x_i$	(C)
45)	If P and p^* are partitions of $[a, b]$ then a partition is called a refinement of partition P if…. (A) $P \subseteq p^*$ (B) $P = p^*$ (C) $P \supseteq p^*$ (D) $p^* = \Phi$	(A)

46)	A bounded function f is integrable on $[a, b]$ if (A) $\underline{\int_a^b} f(x)\mathrm{d}x < \overline{\int_a^b} f(x)\mathrm{d}x$ (B) $\underline{\int_a^b} f(x)\mathrm{d}x > \overline{\int_a^b} f(x)\mathrm{d}x$ (C) $\underline{\int_a^b} f(x)\mathrm{d}x \neq \overline{\int_a^b} f(x)\mathrm{d}x$ (D) $\underline{\int_a^b} f(x)\mathrm{d}x = \overline{\int_a^b} f(x)\mathrm{d}x$	**(D)**
47)	The function $f(x) = 6, \forall x \in [a, b]$ is (A) Riemann integrable on $[a, b]$ (B) Not Riemann integrable on $[a, b]$ (C) Both (A) and (B) (D) None of these	**(A)**
48)	If the function $f(x) = x$ is R-integrable on $[0,1]$ $\int_0^1 x\ \mathrm{d}x = ...$ (A) 0 (B) 1 (C) $\frac{-1}{2}$ (D) $\frac{1}{2}$	**(D)**
49)	A bounded function $f(x)$ is integrable on $[a, b]$ if to every $\varepsilon > 0$ there corresponds $\delta > 0$ such that for every partition P of $[a, b]$ with norm $\mu(P) < \delta$; $U(P, f) - L(P, f)$... (A) $= 0$ (B) < 0 (C) $< \varepsilon$ (D) $> \varepsilon$	**(C)**
50)	If $f(x) = \begin{cases} 0, & if\ x\ is\ rational \\ 1, & if\ x\ is\ irrational \end{cases}$ then (A) $f(x) \in R\ [a, b]$ (B) $f(x) \notin R\ [a, b]$ (C) $f(x)$ is unbounded (D) None of these	**(B)**

51)	If f is a bounded function on $[a, b]$ then to every $\varepsilon > 0$ there corresponds $\delta > 0$ such that for every partition P of $[a, b]$ with norm $\mu(P) < \delta$ (A) $U(P, f) < \overline{\int_a^b} f\,dx + \varepsilon$ (B) $U(P, f) = \overline{\int_a^b} f\,dx + \varepsilon$ (C) $U(P, f) > \overline{\int_a^b} f\,dx + \varepsilon$ (D) None of these	**(A)**
52)	Every …..function on $[a, b]$ is integrable (A) discontinuous (B) unbounded (C) continuous (D) none of these	**(C)**
53)	$\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{n+n}\right]=\cdots$ (A) 0 (B) 1 (C) $\log 2$ (D) $\log 2$	**(D)**